# پیوست ۳

## روابط خیز و شیب در تیرها

### روابط خیز و شیب تیرها

#### خیزها و شیب‌های تیرهای ساده

$v = y$ خیز در جهت

$v' = \frac{dx}{dv}$ = شیب منحنی خیز

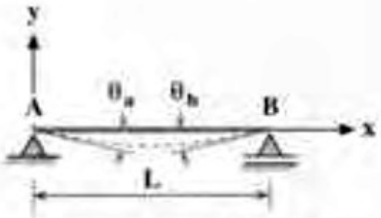
$\delta_c = v\left(\frac{L}{2}\right)$ = خیز در وسط تیر

$x_1$ = فاصله از A تا نقطه خیز ماکزیمم

$\delta_{max} = v_{max}$ = خیز ماکزیمم

$\theta_a = v'(0)$ = شیب در انتهای چپ تیر

$\theta_b = -v'(L)$ = شیب در انتهای راست تیر




ثابت = EI

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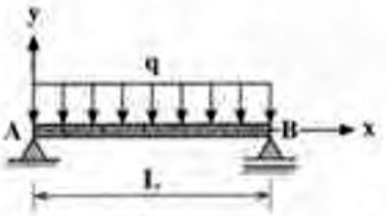
۱.

$$v = \frac{qx}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = \frac{q}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\delta_c = \delta_{max} = \frac{5qL^4}{384EI} \quad \theta_a = \theta_b = \frac{qL^3}{24EI}$$




$$M_{max} = \frac{qL^2}{8} \qquad V_{max} = \frac{qL}{2}$$

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۲.

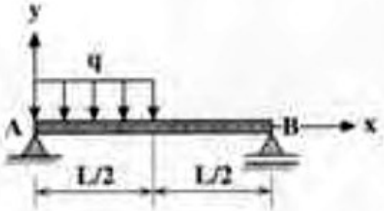
$$v = \frac{qx}{384EI}(9L^3 - 24Lx^2 + 16x^3) \quad 0 \le x \le \frac{L}{2}$$

$$v' = \frac{q}{384EI}(9L^3 - 72Lx^2 + 64x^3) \quad 0 \le x \le \frac{L}{2}$$

$$v = \frac{qL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3) \quad \frac{L}{2} \le x \le L$$

$$v' = \frac{qL}{384EI}(24x^2 - 48Lx + 17L^2) \quad \frac{L}{2} \le x \le L$$

$$\delta_c = \frac{5qL^4}{768EI} \quad \theta_a = \frac{3qL^3}{128EI} \quad \theta_b = \frac{7qL^3}{384EI}$$

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۳.

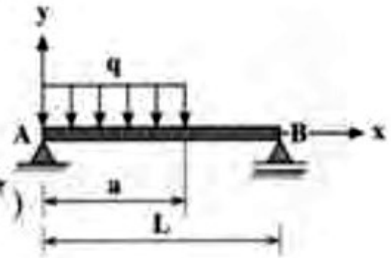
$$v=\frac{qx}{۲۴LEI}(a^۳-۴a^۳L+۴a^۳L^۲+۲a^۳x^۲-۴aLx^۲+Lx^۳) \quad ۰\le x\le a$$

$$v'=\frac{q}{۲۴LEI}(a^۳-۴a^۳L+۴a^۳L^۲+۶a^۳x^۲-۱۲aLx^۲+۴Lx^۳) \quad ۰\le x\le a$$

$$v=\frac{qa^۳}{۲۴LEI}(-a^۳L+۴L^۳x+a^۳x-۶Lx^۲+۲x^۳) \quad a\le x\le L$$

$$v'=\frac{qa^۳}{۲۴LEI}(۴L^۳+a^۳-۱۲Lx+۶x^۲) \quad a\le x\le L$$

$$\theta_a=\frac{qa^۳}{۲۴LEI}(a^۳-۴aL+۴L^۲) \quad \theta_b=\frac{qa^۳}{۲۴LEI}(۲L^۲-a^۳)$$

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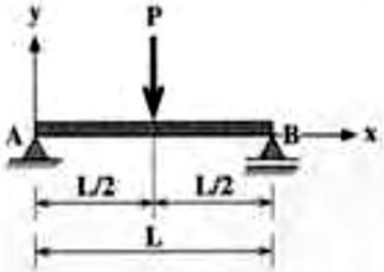
۴.

$$v=\frac{Px}{۴۸EI}(۳L^۲-۴x^۲) \quad ۰\le x\le \frac{L}{۲}$$

$$v'=\frac{P}{۱۶EI}(L^۲-۴x^۲) \quad ۰\le x\le \frac{L}{۲}$$

$$\delta_c=\delta_{max}=\frac{PL^۳}{۴۸EI} \quad \theta_a=\theta_b=\frac{PL^۲}{۱۶EI}$$

$$M_{max}=\frac{PL}{۴} \quad V_{max}=\frac{P}{۲}$$

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۵.

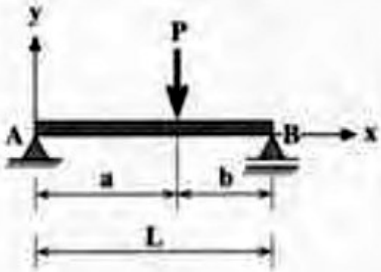
$$v=\frac{Pbx}{۶LEI}(L^۲-b^۲-x^۲) \quad ۰\le x\le a$$

$$v'=\frac{Pb}{۶LEI}(L^۲-b^۲-۳x^۲) \quad ۰\le x\le a$$

$$\theta_a=\frac{Pab(L+b)}{۶LEI} \quad \theta_b=\frac{Pab(L+a)}{۶LEI}$$

$$a\ge b, \delta_c=\frac{Pb(۳L^۲-۴b^۲)}{۴۸EI}$$

$$a\ge b, x_۱=\sqrt{\frac{L^۲-b^۲}{۳}} \quad v_{max}=\frac{Pb(L^۲-b^۲)^{۳/۲}}{۹\sqrt{۳}LEI}$$

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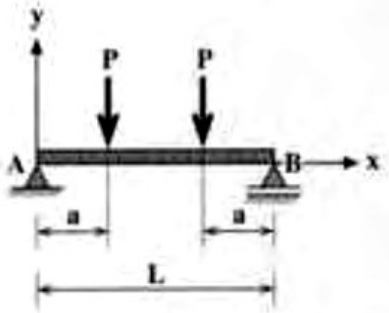
۶.

$$v=\frac{Px}{۶EI}(۳aL-۳a^۲-x^۲) \quad ۰\le x\le a$$

$$v'=\frac{P}{۲EI}(aL-a^۲-x^۲) \quad ۰\le x\le ۲$$

$$v=\frac{Pa}{۶EI}(۳Lx-۳x^۲-a^۲) \quad a\le x\le \frac{L}{۲}$$

$$v'=\frac{Pa}{۲EI}(L-۲x) \quad a\le x\le \frac{L}{۲}$$




$$\theta_a=\frac{Pa(L-a)}{۲EI} \quad \delta_c=v_{max}=\frac{Pa}{۲۴EI}(۳L^۲-۴a^۲) \quad M_{max}=Pa \quad V_{max}=P$$

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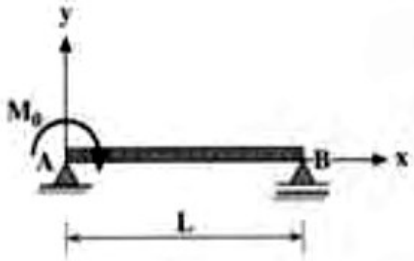
۷.

$$v=\frac{M_۰x}{۶LEI}(۲L^۲-۳Lx+x^۲)$$

$$v'=\frac{M_۰}{۶LEI}(۲L^۲-۶Lx+۳x^۲)$$

$$\delta_c=\frac{M_۰L^۲}{۱۶EI} \quad \theta_a=\frac{M_۰L}{۳EI} \quad \theta_b=\frac{M_۰L}{۶EI}$$

$$x_۱=L\left(۱-\frac{\sqrt{۳}}{۳}\right) \quad \delta_{max}=\frac{M_۰L^۲}{۹\sqrt{۳}EI}$$

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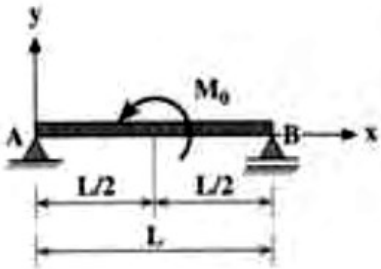
۸.

$$v=\frac{M_۰x}{۲۴LEI}(L^۲-۴x^۲) \quad ۰\le x\le \frac{L}{۲}$$

$$v'=\frac{M_۰}{۲۴LEI}(L^۲-۱۲x^۲) \quad ۰\le x\le \frac{L}{۲}$$

$$\delta_c=۰ \quad \theta_a=\frac{M_۰L}{۲۴EI} \quad \theta_b=-\frac{M_۰L}{۲۴EI}$$

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۹.

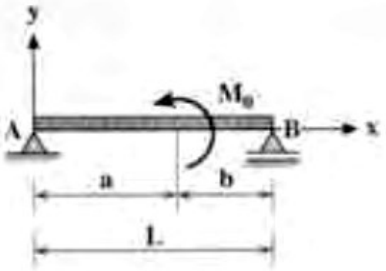
$$v=\frac{M_0 x}{6LEI}(6aL-3a^2-2L^2-x^2) \quad 0\le x\le a$$

$$v'=\frac{M_0}{6LEI}(6aL-3a^2-2L^2-3x^2) \quad 0\le x\le a$$

$$v=\frac{M_0 a}{3LEI}(3aL-2a^2-L^2) \quad : \quad x=a \text{ در}$$

$$v'=\frac{M_0}{3LEI}(3aL-3a^2-L^2) \quad : \quad x=a \text{ در}$$

$$\theta_a=\frac{M_0}{6LEI}(6aL-3a^2-2L^2) \qquad \theta_b=\frac{M_0}{6LEI}(3a^2-L^2)$$

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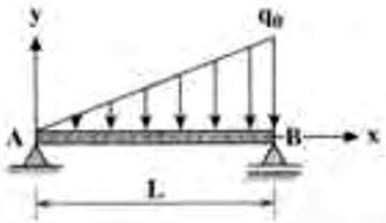
۱۰.

$$v=\frac{q_0 x}{360LEI}(7L^4-10L^2x^2+3x^4)$$

$$v'=\frac{q_0}{360LEI}(7L^4-30L^2x^2+15x^4)$$

$$\delta_c=\frac{5q_0L^4}{768EI} \qquad \theta_a=\frac{7q_0L^3}{360EI} \qquad \theta_b=\frac{q_0L^3}{45EI}$$

$$x_1=0/5193L \qquad \delta_{max}=0/00652\frac{q_0L^4}{EI} \qquad M_{max}=\frac{q_0L^2}{15/6} \qquad V_{max}=\frac{q_0L}{3}$$

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**روابط خیز و شیب در تیرهای طره‌ای:**

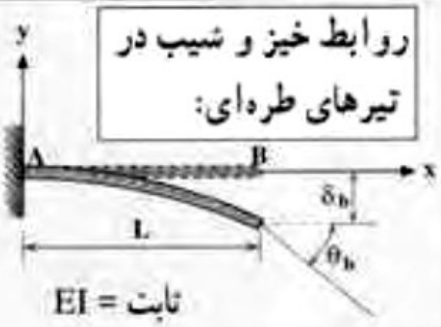
$v=y$ خیز در جهت

$v'=\frac{dv}{dx}$ شیب منحنی خیز

$\delta_b=v(L)=$ خیز در انتهای راست تیر

$\theta_b=v'(L)=$ شیب در انتهای راست تیر

$EI=$ ثابت

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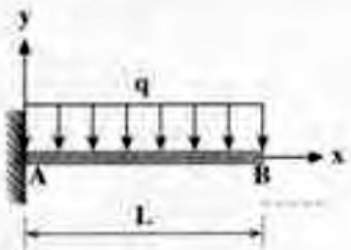
۱۱.

$$v=\frac{qx^2}{24EI}(6L^2-4Lx+x^2)$$

$$v'=\frac{qx}{6EI}(3L^2-3Lx+x^2)$$

$$\delta_b=\frac{qL^4}{8EI} \qquad \theta_b=\frac{qL^3}{6EI}$$

$$M_{max}=\frac{qL^2}{2} \qquad V_{max}=qL$$

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۲.

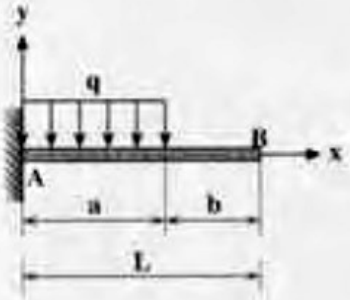
$$v=\frac{qx^2}{24EI}(6a^2-4ax+x^2) \quad 0\le x\le a$$

$$v'=\frac{qx}{6EI}(3a^2-3ax+x^2) \quad 0\le x\le a$$

$$v=\frac{qa^3}{24EI}(4x-a) \quad v'=\frac{qa^3}{6EI} \quad a\le x\le L$$

$$v=\frac{qa^4}{8EI} \quad v'=\frac{qa^3}{6EI} \quad x=a \text{ در}$$

$$\delta_b=\frac{qa^3}{24EI}(4L-a) \quad \theta_b=\frac{qa^3}{6EI}$$

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۳.

$$v=\frac{qx^2}{12EI}(3bL+3ab-2bx) \quad 0\le x\le a$$

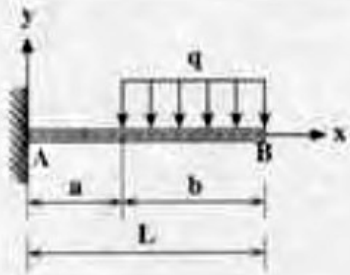
$$v'=\frac{qbx}{2EI}(L+a-x) \quad 0\le x\le a$$

$$v=\frac{q}{24EI}(x^4-4Lx^3+6L^2x^2-4a^3x+a^4) \quad a\le x\le L$$

$$v'=\frac{q}{6EI}(x^3-3Lx^2+3L^2x-a^3) \quad a\le x\le L$$

$$v=\frac{qa^2b}{12EI}(3L+a) \quad v'=\frac{qabL}{2EI} \quad x=a \text{ در}$$

$$\delta_b=\frac{q}{24EI}(3L^4-4a^3L+a^4) \quad \theta_b=\frac{q}{6EI}(L^3-a^3)$$

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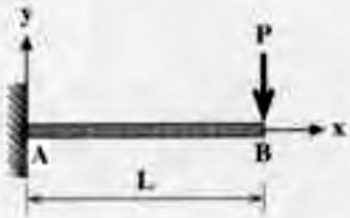
۴.

$$v=\frac{Px^2}{6EI}(3L-x) \quad v'=\frac{Px}{2EI}(2L-x)$$

$$\delta_b=\frac{PL^3}{3EI} \quad \theta_b=\frac{PL^2}{2EI}$$

$$M_{max}=PL \quad V_{max}=P$$

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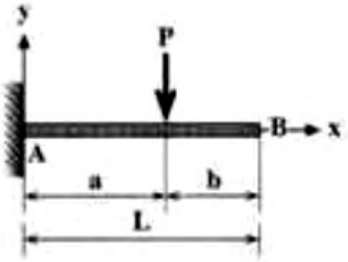
۵.

$$v=\frac{Px^2}{6EI}(3a-x) \qquad v'=\frac{Px}{2EI}(2a-x) \qquad 0\le x\le a$$

$$v=\frac{Pa^2}{6EI}(3x-a) \qquad v'=\frac{Pa^2}{2EI} \qquad a\le x\le L$$

$$v=\frac{Pa^3}{3EI} \qquad v'=\frac{Pa^2}{2EI} \qquad x=a \text{ در}$$

$$\delta_b=\frac{Pa^2}{6EI}(3L-a) \qquad \theta_b=\frac{Pa^2}{2EI}$$

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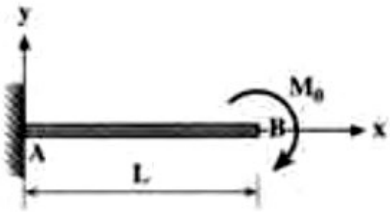
۶.

$$v=\frac{M_0x^2}{2EI} \qquad v'=\frac{M_0x}{EI}$$

$$\delta_b=\frac{M_0L^2}{2EI} \qquad \theta_b=\frac{M_0L}{EI}$$

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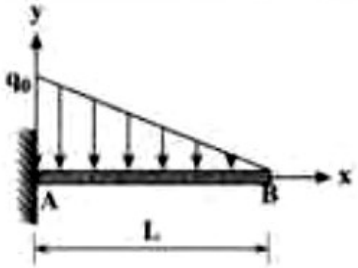
۷.

$$v=\frac{q_0x^2}{120LEI}(10L^3-10L^2x+5Lx^2-x^3)$$

$$v'=\frac{q_0x}{24LEI}(4L^3-6L^2x+4Lx^2-x^3)$$

$$\delta_b=\frac{q_0L^4}{30EI} \qquad \theta_b=\frac{q_0L^3}{24EI} \qquad M_{max}=\frac{q_0L^2}{6} \qquad V_{max}=\frac{q_0L}{2}$$

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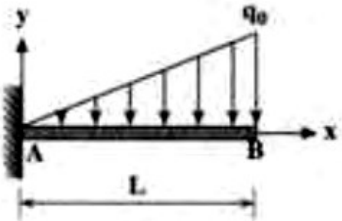
۸.

$$v=\frac{q_0x^2}{120LEI}(20L^3-10L^2x+x^3)$$

$$v'=\frac{q_0x}{24LEI}(8L^3-6L^2x+x^3)$$

$$\delta_b=\frac{11q_0L^4}{120EI} \qquad \theta_b=\frac{q_0L^3}{8EI}$$

$$M_{max}=\frac{q_0L^2}{3} \qquad V_{max}=\frac{q_0L}{2}$$

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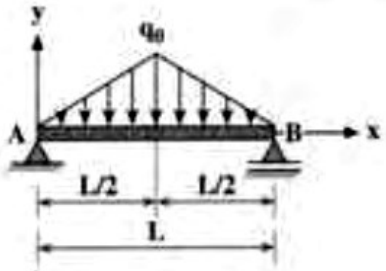
$$\delta_{max} = \frac{q.l^4}{120EI}$$

$$M_{max} = \frac{q.L^2}{12} \qquad V_{max} = \frac{q.L}{4}$$




$$\delta_{max} = \frac{3q.l^4}{640EI}$$

$$M_{max} = \frac{q.L^2}{24} \qquad V_{max} = \frac{q.L}{4}$$